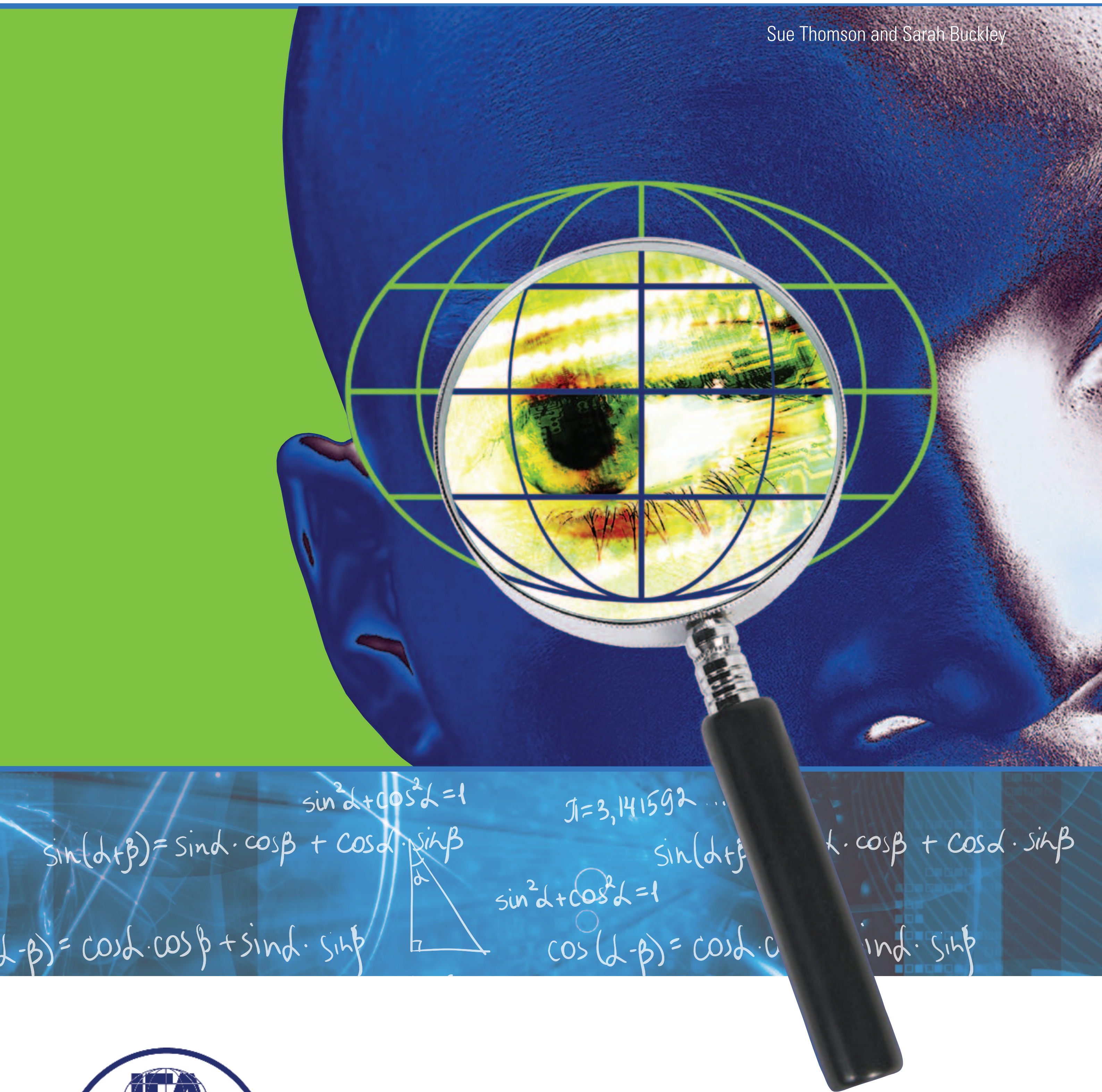
Informing mathematics pedagogy: TIMSS 2007

Australia and the world

Sue Thomson and Sarah Buckley



IEA TIMSS 2007

A further investigation from the Trends in International Mathematics and Science Study (TIMSS) 2007

ACER

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Contents

This report

This report aims to provide teachers with more detailed information on what Australian students are actually able to do in mathematics in terms of the TIMSS assessment. A detailed assessment of Australian students' performance can be found in the full report that was written to inform the educational community about Australian students' performance on national and international scales. The report - *TIMSS 07: Taking a closer look at mathematics and science in Australia* - and more information about TIMSS can be accessed at www.acer.edu.au/timss.

What is TIMSS?

TIMSS is a key part of MCEETYA's National Assessment Program.

TIMSS is the Trends in International Mathematics and Science Study. TIMSS 2007 was the fourth in a cycle of internationally comparative assessments, conducted by the International Association for the Evaluation of Educational Achievement (IEA). The IEA is dedicated to improving teaching and learning in mathematics and science for students around the world.

Carried out every four years with Year 4 and Year 8 students, TIMSS provides data about national and international trends in mathematics and science achievement. In Australia, TIMSS is part of the Ministerial Council on Education, Employment, Training and Youth Affairs' (MCEETYA) National Assessment Program. It provides a level of international benchmarking to complement national assessments at Years 3,5,7 and 9 and other sample-based national studies.

To inform educational policy in the participating countries, TIMSS also routinely collects extensive background information that addresses concerns about the quantity, quality, and content of instruction.

What is the focus of TIMSS?

The main goal of TIMSS is to assist countries to monitor and evaluate their mathematics and science teaching across time and across year levels.

TIMSS examines three levels of the curriculum: what is intended to be taught, what is actually taught, and what it is that students learn!

TIMSS has a curriculum focus. The three levels of the curriculum defined by TIMSS are:

The *intended* curriculum – the curriculum as specified at national or system level.

- What are mathematics and science students around the world expected to learn?
- How do countries vary in their intended goals, and
- What characteristics of education systems, schools and students influence the development of these goals?

The *implemented* curriculum – the curriculum as interpreted and delivered by classroom teachers.

- What opportunities are provided for students to learn mathematics and science?
- How do instructional practices vary among countries, and
- What factors influence these variations?

The *attained* curriculum – which is that part of the curriculum that is learned by students, as demonstrated by their attitudes and achievements.

- What mathematics and science concepts, processes and attitudes have students learned?
- What factors are linked to students' opportunity to learn, and
- How do these factors influence students' achievements?

What do participants have to do?

Students complete an assessment booklet that contains an equal number of questions about mathematics and science. Questions are presented in two general formats: multiple choice and constructed response. After the students complete the assessment booklet they complete a short questionnaire. Teachers and principals also complete questionnaires.

Students complete an assessment and a background questionnaire. Teachers and principals also complete a questionnaire.

These internationally standard questionnaires gather information at the student, class, and school level. The student questionnaire gathers information from students about their family background, aspects of learning and instruction in mathematics and science, and the context of instruction. The teacher questionnaire collects information about a variety of issues related to qualifications, pedagogical practices, teaching styles, use of technology, assessment and assignment of homework, and classroom climate. The school questionnaire, completed by the principal, gathers descriptive information about the school and information about instructional practices. For example, questions were asked about recruitment of teachers and numbers of staff, teacher morale, school and teacher autonomy, school resources, and school policies and practices such as the use of student assessments.

Who participated in TIMSS 2007?

Internationally

A total of 49 countries at Year 8 and 36 countries at Year 4 participated in TIMSS 2007. In addition, four provinces of Canada, two states of the United States, Dubai Emirate, UAE and Basque Country, Spain, were also in the study as benchmarking participants*. These are shown in Figure 1.

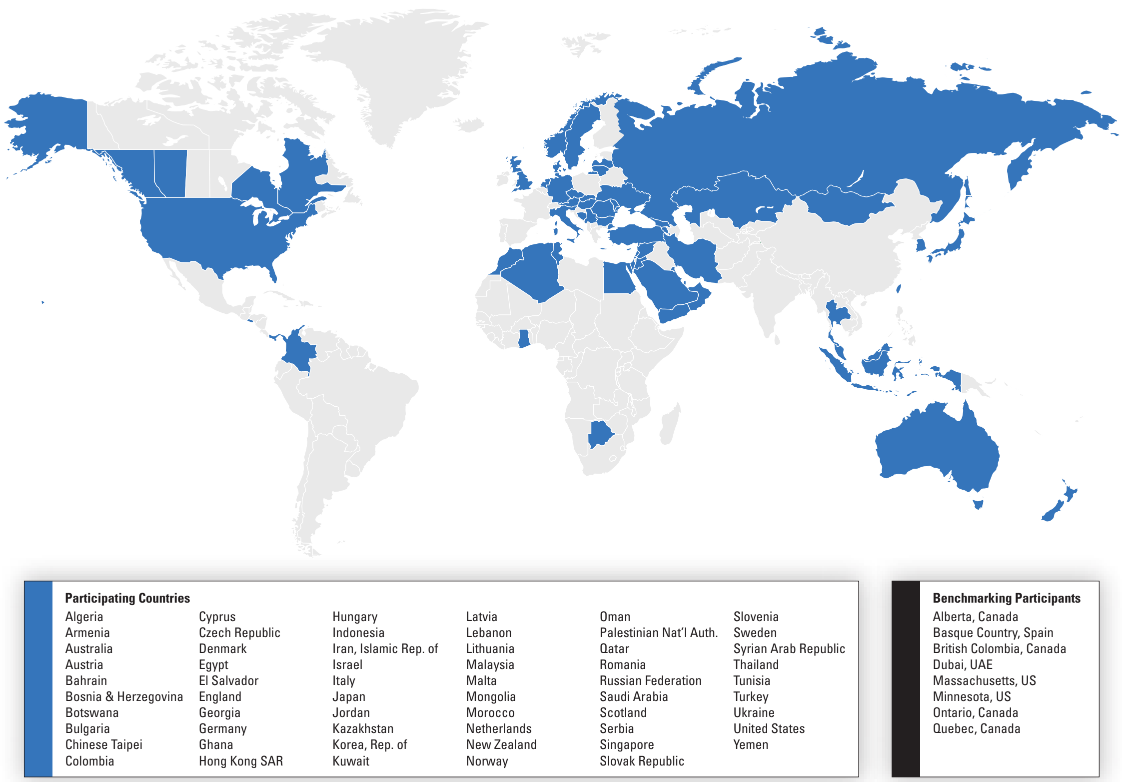


Figure 1 Map of participating countries

* Benchmarking participants are provinces or regions that participated in TIMSS for their own internal benchmarking purposes. Data from these regions are not included in the international average.

In Australia

Schools are chosen from all schools in Australia to be representative of their state and sector.

A stratified random sample of 230 primary schools and 230 secondary schools was chosen in Australia, and of this sample, 229 primary schools and 228 secondary schools participated in the data collection for TIMSS 2007. The sample is drawn from all schools in Australia, and is representative of all states and sectors. In each state, government, Catholic and independent schools are chosen proportional to their number in the state. Figure 2 shows all schools in Australia (including those on Christmas Island, Norfolk Island and King Island) in blue and all schools selected for TIMSS in black.

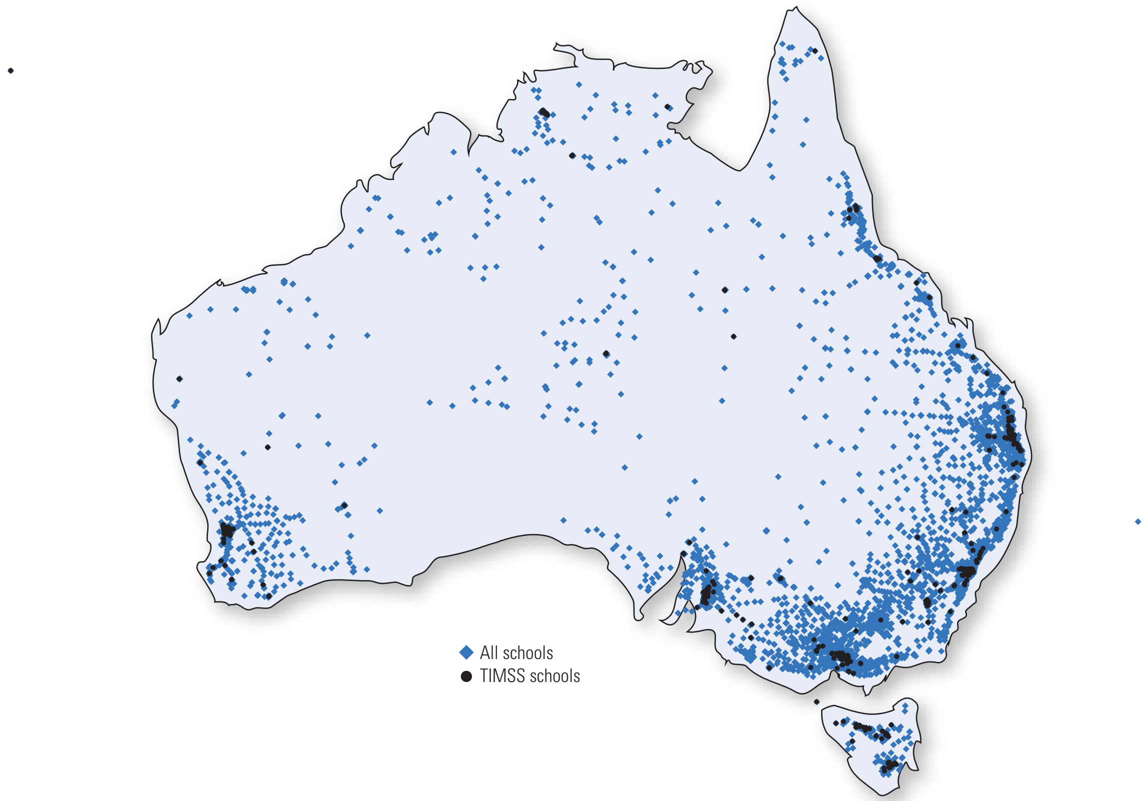


Figure 2 Australian schools and TIMSS sample schools

How is mathematics assessed in TIMSS?

TIMSS is curriculum based. All countries are surveyed to ensure that the test is kept up-to-date.

The assessment has both a content dimension and a cognitive dimension.

At the beginning of each TIMSS cycle, a committee comprised of curriculum experts in mathematics and science from a range of participating countries reviews the framework for the upcoming assessment. This framework is finally ratified by all member countries as being representative of their country's curricula. For TIMSS to not become a lowest common denominator assessment, however, there will always be some content that is not covered in the curriculum of each country. This is managed at the data analysis stage of the project by removing the items that a country argues is outside their curriculum. It rarely makes any difference to a country's score.

The general framework for the TIMSS 2007 assessment of mathematics had two dimensions, one relating to context and the other relating to cognition. Within the content dimension, there were three domains in mathematics at Year 4 and four at Year 8. In addition there were three cognitive domains in each curriculum area: *knowing*, *applying* and *reasoning*. The two dimensions and their domains were the foundation of the mathematics assessments. The content domains defined the specific subject matter covered by the assessment, and the cognitive domains defined the sets of skills expected of students as they engage with the content. An elaboration of the content domains is shown below in Table 1 and Table 2, and includes proportions of each topic area examined in the TIMSS tests.

Table 1 TIMSS content domains in mathematics at Year 4

Mathematics Year 4	
Content Domains	**Topic areas**
Number (50%)	Whole numbers
	Fractions and decimals
	Number sentences
	Patterns and relationships
Geometric shapes and measurement (35%)	Lines and angles
	Two- and three-dimensional shapes
	Location and movement
Data display (15%)	Reading and interpreting
	Organising and representing

Table 2 TIMSS content domains in mathematics at Year 8

Mathematics Year 8	
Content Domains	**Topic areas**
Number (30%)	Whole numbers
	Fractions and decimals
	Integers
	Ratio, proportion and per cent
Algebra (30%)	Patterns
	Algebraic expressions
	Equations/formulas and functions
Geometry (20%)	Geometric shapes
	Geometric measurement
	Location and movement
Data and Chance (20%)	Data organisation and representation
	Data interpretation
	Chance

The structure of the TIMSS assessment

A consequence of the assessment goals of TIMSS is that there are many more questions on the assessment than can be answered by a student in the amount of testing time available. To work around this, TIMSS uses an approach that involves packaging the entire assessment pool of mathematics and science questions into a set of 14 student achievement booklets, with each student completing just one booklet.

Each question, or item, appears in two booklets, providing a mechanism for linking together the student responses from the various booklets. Booklets are distributed among students in participating classrooms so that the groups of students completing each booklet are approximately equivalent in terms of student ability.

Using Item-Response Theory (IRT) scaling techniques, a comprehensive picture of the achievement of the entire student population is assembled from the combined responses of individual students to the booklets they are assigned. This approach reduces to manageable proportions what otherwise would be an impossible student burden, albeit at the cost of greater complexity in booklet assembly, data collection, and data analysis.

There are 14 different student achievement test booklets – these all form part of the jigsaw that we put together to enable the sample to represent all Australian students.

A number of new items are developed for each new TIMSS assessment. In addition, because TIMSS is an assessment that examines trends, a number of items are retained from one cycle to the next, to link assessments to each other.

Question types and scoring the responses

Two question formats are used in the TIMSS assessment – multiple-choice and constructed-response. At least half of the total number of points represented by all the questions will come from multiple-choice questions. Each multiple-choice question is worth one score point.

Multiple-Choice Questions. Multiple-choice questions provide four response options, of which only one is correct. These questions can be used to assess any of the behaviours in the cognitive domains. However, because they do not allow for students' explanations or supporting statements, multiple-choice questions may be less suitable for assessing students' ability to make more complex interpretations or evaluations.

In assessing Year 4 and Year 8 students, it is important that linguistic features of the questions be developmentally appropriate. Therefore, the questions are written clearly and concisely. The response options also are written succinctly in order to minimise the reading load of the question. The options that are incorrect are written to be plausible, but not deceptive. For students who may be unfamiliar with this test question format, the instructions given at the beginning of the test include a sample multiple-choice item that illustrates how to select and mark an answer.

Constructed-Response Questions. For this type of test item students are required to construct a written response, rather than select a response from a set of options. Constructed-response questions are particularly well-suited for assessing aspects of knowledge and skills that require students to explain phenomena or interpret data based on their background knowledge and experience.

The scoring guide for each constructed-response question describes the essential features of appropriate and complete responses. The guides focus on evidence of the type of behaviour the question assesses. They describe evidence of partially correct and completely correct responses. In addition, sample student responses at each level of understanding provide important guidance to those who will be rating the students' responses. In scoring students' responses to constructed-response questions, the focus is solely on students' achievement with respect to the topic being assessed, not on their ability to write well. However, students need to communicate in a manner that will be clear to those scoring their responses.

How results are reported in TIMSS

TIMSS summarises achievement for each year level in two ways. Firstly, results are reported on a scale with a mean of 500 and a standard deviation of 100. However, it should be noted that the results for year 4 and year 8 are not directly comparable. While the scales for the two year levels are expressed in the same numerical units, they are not directly comparable in terms of being able to say how much achievement or learning at one year level equals how much achievement or learning is observed in the other year level. That is, achievement cannot be described at either year level in absolute terms.

International Benchmarks

Another way in which general achievement was assessed in TIMSS was by examining the percentage of students in each country that reached certain benchmarks. While the achievement scales mentioned in the previous section summarise student performance in the cognitive and content knowledge measured by the TIMSS mathematics tests, the international benchmarks help put these scores in context.

Internationally, it was decided that performance should be measured at four levels. These four levels summarise the achievement reached at:

- the 'advanced international benchmark', which was set at a score of 625;
- the 'high international benchmark', which was set at a score of 550;
- the 'intermediate international benchmark', which was set at a score of 475; and
- the 'low international benchmark', which was set at a score of 400.

The benchmarks discussed in this report were based solely on student performance in TIMSS 2007. It should also be noted that when reporting the percentage of students achieving a particular benchmark, this includes students achieving the benchmarks above this. For example, if 24 per cent of Year 8 students achieved the high international benchmark this would include the six per cent at the advanced benchmark.

Year 4 Mathematics – Descriptors of performance at the international benchmarks

In Year 4 mathematics, students at the Advanced International Benchmark were able to apply mathematical understanding and knowledge in a variety of relatively complex problem situations and were able to explain their reasoning, whereas those at the Low International Benchmark demonstrated some basic mathematical knowledge and were able to compute with whole numbers, recognize some geometric shapes, and read simple graphs and tables. Table 3 gives some brief descriptors of achievement at the international benchmarks for Year 4 mathematics, and following this, an example is provided for each of the levels.

Table 3 Descriptors for Year 4 mathematics international benchmarks

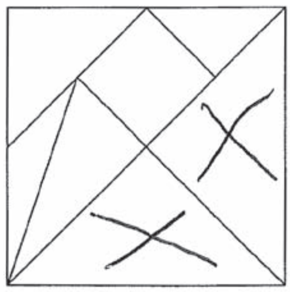
Year 4	Low International Benchmark	Intermediate International Benchmark	High International Benchmark	Advanced International Benchmark
	(400)	(475)	(550)	(625)
	Students have some basic mathematical knowledge. Students demonstrate an understanding of adding and subtracting with whole numbers. They demonstrate familiarity with triangles and informal coordinate systems. They can read information from simple bar graphs and tables.	*Students can apply basic mathematical knowledge in straightforward situations.* Students at this level demonstrate an understanding of whole numbers. They can extend simple numeric and geometric patterns. They are familiar with a range of two-dimensional shapes. They can read and interpret different representations of the same data.	*Students can apply their knowledge and understanding to solve problems.* Students can solve multi-step word problems involving operations with whole numbers. They can use division in a variety of problem situations. They demonstrate understanding of place value and simple fractions. Students can extend patterns to find a later specified term and identify the relationship between ordered pairs. Students show some basic geometric knowledge. They can interpret and use data in tables and graphs to solve problems.	*Students can apply their understanding and knowledge in a variety of relatively complex situations and explain their reasoning.* They can apply proportional reasoning in a variety of contexts. They demonstrate a developing understanding of fractions and decimals. They can select appropriate information to solve multi-step word problems. They can formulate or select a rule for a relationship. Students can apply geometric knowledge of a range of two- and three-dimensional shapes in a variety of situations. They can organise, interpret, and represent data to solve problems.

Year 4 Mathematics: Performance at the Low International Benchmark

In this example, students were asked to use their knowledge of shape and size to classify and identify which of the triangles in the diagram were the same. This is an example of the type of item likely to be answered correctly by students reaching the low international benchmark.

Internationally, 72 per cent of students correctly identified the two triangles; however this was as high as 91 per cent in Hong Kong and Slovenia. In Australia, 85 per cent of students answered correctly, which was significantly higher than the international average.

The square is cut into 7 pieces. Put an X on each of the 2 triangles that are the same size and shape.

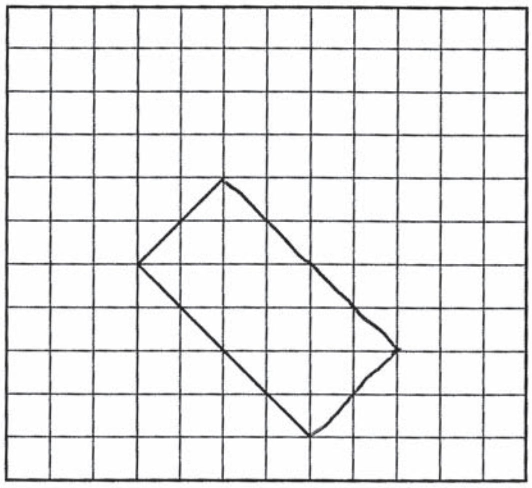


Country	Per cent full correct
Hong Kong	91 (1.2)
Slovenia	91 (1.3)
Lithuania	89 (1.3)
England	88 (1.4)
Australia	85 (1.9)
United States	85 (1.0)
New Zealand	81 (1.4)
International average	**72 (0.3)**
Yemen	13 (1.5)

Year 4 Mathematics: Performance at the Intermediate International Benchmark

In this example, from the domain of *geometric shapes and measures*, students were given two adjacent sides of a rectangle on a grid and asked to draw the other two sides. On average internationally more than half of the students completed the rectangle correctly. In Hong Kong 90 per cent of students answered correctly and Australian students also did well, with 68 per cent completing the rectangle correctly.

Here are two sides of a rectangle. Draw the other two sides.



Country	Per cent full correct
Hong Kong	90 (1.4)
Japan	78 (1.8)
Chinese Taipei	77 (1.9)
England	70 (1.9)
Australia	68 (3.3)
New Zealand	61 (1.8)
United States	55 (1.7)
International average	**54 (0.4)**
Yemen	5 (1.0)

Year 4 Mathematics: Performance at the High International Benchmark

This constructed-response item, involving subtraction with three digits, shows the type of item generally answered correctly by students at the high international benchmark. This item was answered correctly by 42 per cent of students internationally, and by 88 per cent of students in Chinese Taipei. In ten countries internationally (Chinese Taipei, Hong Kong, Singapore, Russian Federation, Kazakhstan, Japan, Lithuania, Latvia, Ukraine and Armenia) two-thirds or more students answered this item correctly, however in Australia only 20 per cent of students did so.

$$\begin{array}{r} 942 \\ -5\bullet 7 \\ \hline 415 \end{array}$$

Adam did the subtraction problem above for homework but spilled some of his drink on it. One digit could not be read. His answer of 415 was correct. What is the missing digit?

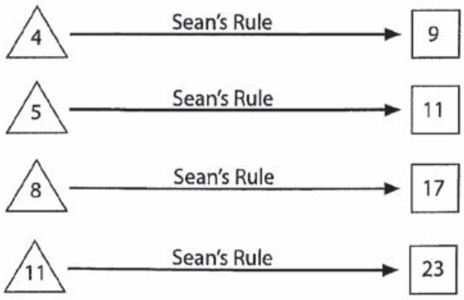
Answer: 2

Country	Per cent full correct
Chinese Taipei	88 (1.6)
Hong Kong	85 (1.9)
Singapore	85 (1.4)
International average	**42 (0.4)**
United States	41 (1.8)
England	28 (2.1)
Australia	20 (3.1)
New Zealand	18 (1.6)
Qatar	5 (0.8)

Year 4 Mathematics: Performance at the Advanced International Benchmark

At the Year 4 level, pre-algebraic concepts and skills are a part of the TIMSS framework and assessment. Students at this age typically explore number patterns, investigate the relationships between the terms and find or use the rules that generate them. The following example shows a number pattern item likely to be answered correctly by students who are performing at the advanced benchmark.

In this item students were shown a linear relationship between pairs of numbers and asked to write the two-step rule that described how to get the second number from the first. Internationally, 15 per cent of students were able to provide a correct response to this item. In Australia 20 per cent answered correctly, however in Hong Kong, Japan and Singapore the proportion was between 36 and 39 per cent.



Sean used the same rule to get the number in the □ from the number in the △.
What was the rule?

Answer: double the number and add 1

Country	Per cent full correct
Hong Kong	39 (2.7)
Japan	38 (2.1)
Singapore	36 (2.1)
England	28 (2.3)
USA	23 (1.4)
Australia	20 (3.1)
International average	**15 (0.3)**
New Zealand	17 (1.6)
El Salvador	0 (0.0)

Year 8 Mathematics – Descriptors of performance at the international benchmarks

At Year 8, students at the Advanced International Benchmark organised and drew conclusions from information, made generalisations, and solved non-routine problems involving numeric, algebraic, and geometric concepts and relationships. In comparison, those at the Low International Benchmark demonstrated some knowledge of whole numbers and decimals, operations, and basic graphs.

Table 4 provides descriptors for each of the benchmarks at Year 8 level, and this table is followed by examples of each level of the benchmark.

Table 4 Descriptors for Year 8 mathematics international benchmarks

Year 8	Low International Benchmark	Intermediate International Benchmark	High International Benchmark	Advanced International Benchmark
	(400)	(475)	(550)	(625)
	Students have some knowledge of whole numbers and decimals, operations, and basic graphs.	*Students can apply basic mathematical knowledge in straightforward situations.* They can add and multiply to solve one-step word problems involving whole numbers and decimals. They can work with familiar fractions. They understand simple algebraic relationships. They demonstrate understanding of properties of triangles and basic geometric concepts. They can read and interpret graphs and tables. They recognise basic notions of likelihood.	*Students can apply their understanding and knowledge in a variety of relatively complex situations.* They can relate and compute with fractions, decimals, and percentages, operate with negative integers, and solve word problems involving proportions. Students can work with algebraic expressions and linear equations. Students use knowledge of geometric properties to solve problems, including area, volume, and angles. They can interpret data in a variety of graphs and tables and solve simple problems involving probability.	*Students can organise and draw conclusions from information, make generalisations, and solve non-routine problems.* They can solve a variety of ratio, proportion, and percentage problems. They can apply their knowledge of numeric and algebraic concepts and relationships. Students can express generalisations algebraically and model situations. They can apply their knowledge of geometry in complex problem situations. Students can derive and use data from several sources to solve multi-step problems.

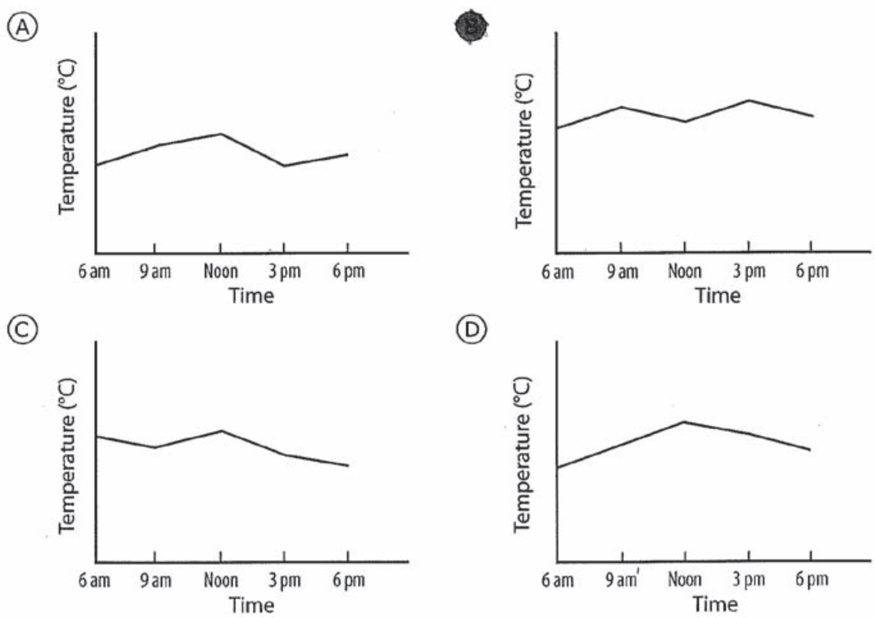
Year 8 Mathematics: Performance at the Low International Benchmark

In this example, students are expected to be able to draw on their knowledge in the *data and chance* domain to match the data in a line graph with the data in a table. The temperatures in the table rise and fall across time, and students needed to recognise that only one graph has this up and down pattern. Seventy-two per cent of students, internationally, answered this item correctly. At least 90 per cent of students in Korea, Japan, Singapore, Chinese Taipei, Lithuania and Slovenia also answered correctly, and 87 per cent of Australian students also answered correctly, significantly higher than the international average.

The table shows the temperatures at various times on a certain day.

Time	6 am	9 am	Noon	3 pm	6 pm
Temperature °C	12	17	14	18	15

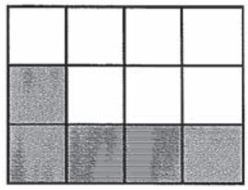
A graph, without a temperature scale, is drawn. Of the following, which could be the graph that shows the information given in the table?



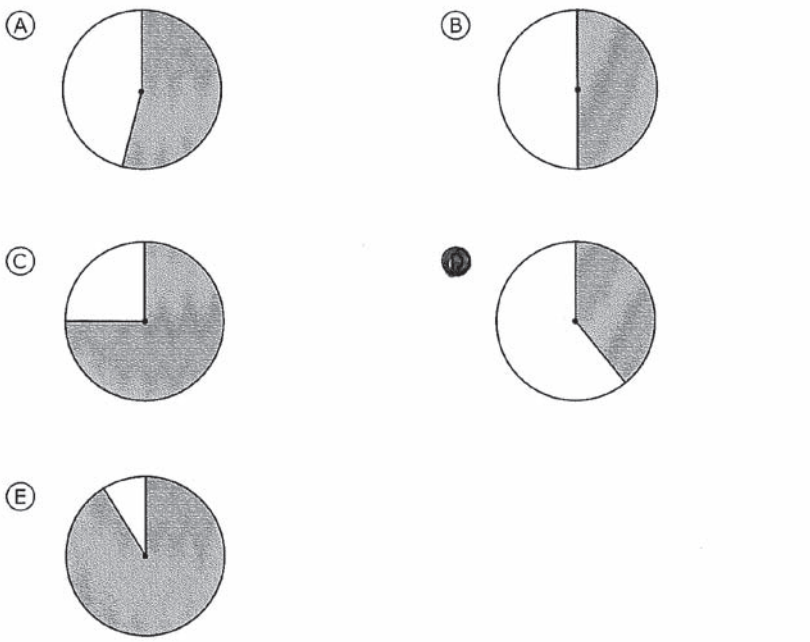
Country	Per cent full correct
Korea	97 (0.7)
Japan	96 (0.8)
Singapore	93 (1.1)
United States	89 (1.0)
Australia	87 (1.7)
England	81 (2.1)
International average	**72 (0.3)**
Qatar	40 (1.6)

Year 8 Mathematics: Performance at the Intermediate International Benchmark

This example shows the type of item that was typically answered correctly by students at the intermediate benchmark. This item called on students' understanding of representations of fractions. Students needed to recognize that of the circular models presented, only the one showing less than ½ best represents the fractional part shown in a rectangle as 5/12. On average internationally, 63 per cent of the Year 8 students answered correctly. The Korean students were the top-performers with 89 per cent answering correctly. Students in Australia also performed well on this item, with three-quarters answering correctly, significantly higher than the international average.



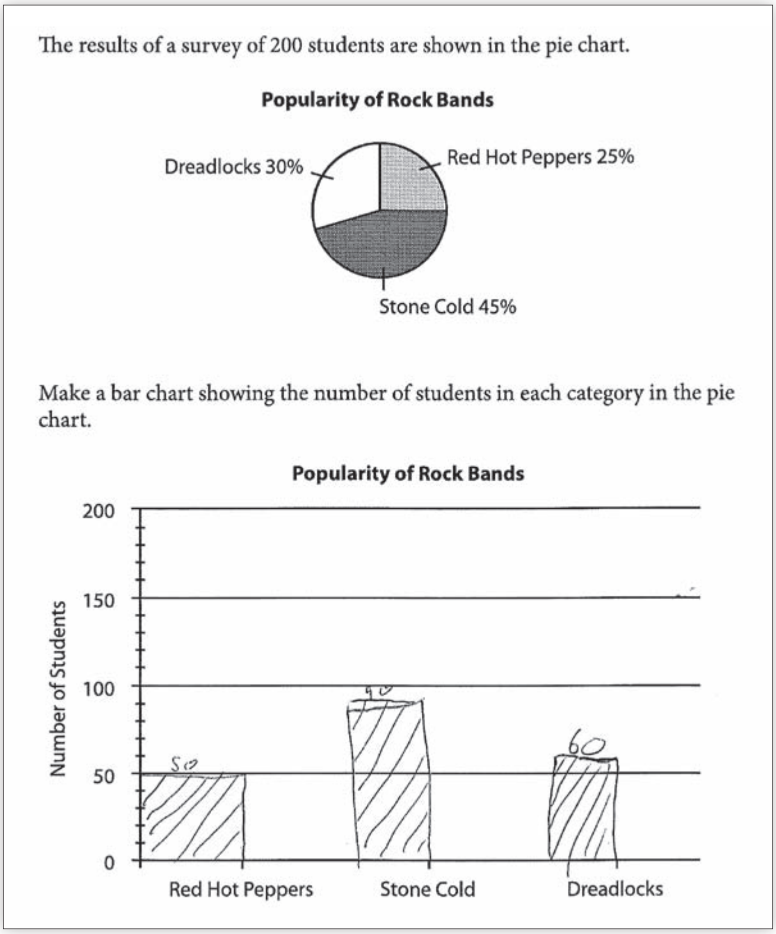
Which circle has approximately the same fraction of its area shaded as the rectangle above?



Country	Per cent full correct
Korea	89 (1.3)
Japan	85 (1.8)
Hong Kong	82 (2.3)
United States	81 (2.2)
England	77 (2.2)
Australia	75 (2.3)
International average	**63 (0.3)**
Ghana	34 (2.3)

Year 8 Mathematics: Performance at the High International Benchmark

This example presents an item from the *data and chance* domain which assesses students' ability to read, organise and display data using various types of graphs, in this case a bar graph and a pie chart. Students needed to draw the bar graph in its entirety to receive full credit, and 27 per cent of students internationally received full credit for this item. In the Asian countries of Korea, Singapore, Chinese Taipei, Japan and Hong Kong, at least two-thirds of students gained full credit on this item, and the proportion of students gaining full credit in Australia (38%), the United States (40%) and England (45%) was also significantly higher than the international average.

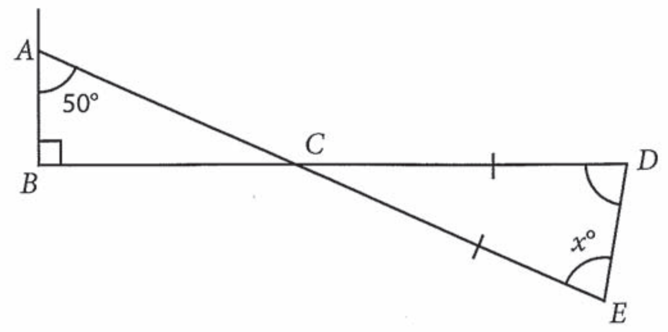


Country	Per cent full correct
Korea	76 (2.0)
Singapore	75 (1.7)
Chinese Taipei	70 (2.1)
England	45 (2.7)
United States	40 (1.9)
Australia	38 (2.7)
International average	**27 (0.3)**
Ghana	2 (0.6)

Year 8 Mathematics: Performance at the Advanced International Benchmark

Students at the advanced benchmark demonstrated fluency with many of the most complex topics in the mathematics framework. This item is from the *geometry* domain, and asks students to use the properties of isosceles and right-angled triangles to find the size of an angle.

Around three-quarters of the students in Singapore, Chinese Taipei, Korea and Japan responded correctly to this item, but only around one-third (32%) of students in Australia and internationally answered it correctly.



In this diagram, $CD = CE$.
What is the value of x?

- A 40
- B 50
- C 60
- 70

Country	Per cent full correct
Singapore	75 (1.7)
Chinese Taipei	73 (2.2)
Korea	73 (1.8)
Japan	71 (1.9)
England	42 (2.8)
International average	**32 (0.3)**
Australia	32 (2.8)
United States	26 (1.4)
Ghana	14 (1.5)

International results on the TIMSS mathematics assessment

To place students' responses in a wider context, the item breakdown presented in this report for Australian students was compared with the responses from students in other countries. Two countries were chosen for this international comparison. The first was Chinese-Taipei. Chinese-Taipei consistently performed in the top three of the 36 countries that participated at Year 4, and the 49 countries that participated at Year 8. Comparison with these students' responses provided an 'upper benchmark' for Australian students. The second country chosen was the United States. The US and Australia are often compared to one another because of curriculum and general cultural similarities.

Figure 3 shows TIMSS 2007 average levels of achievement for mathematics in Years 4 and 8 for Australia, Chinese-Taipei and the US. In both figures, the TIMSS scale average is 500. The box below shows how to read the graphs.

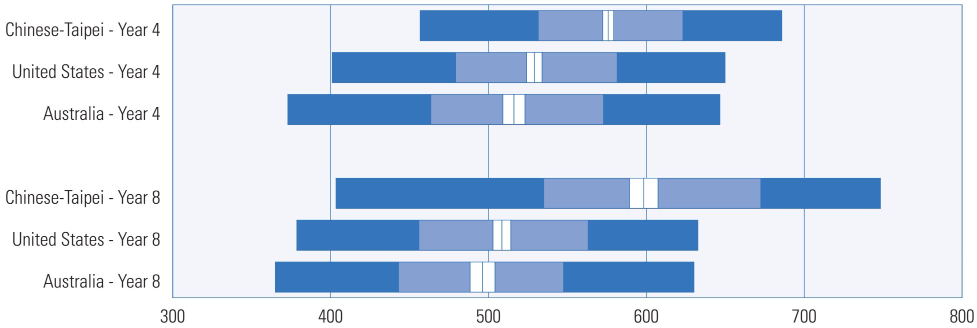
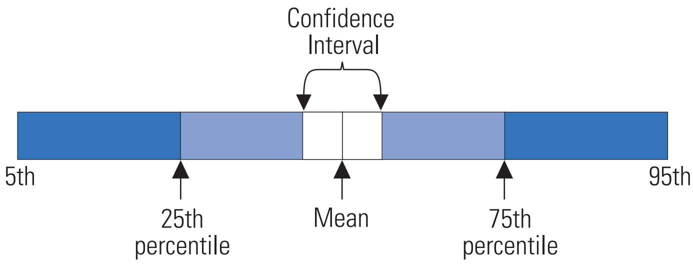


Figure 3 TIMSS 2007 achievement in Year 4 and Year 8 mathematics

Figure 3 shows that Australia consistently ranked behind Chinese-Taipei and the United States for Year 4 and Year 8 mathematics. The average mathematics achievement of Australian Year 4 students was statistically higher than the TIMSS scale average of 500. Compared with all participating countries, Australia's Year 4 performance was significantly higher than 20 countries, similar to three countries but below that of 12 countries, including the US and Chinese-Taipei. Figure 3 also reveals that the average achievement for Australian Year 8 students was below the TIMSS average although the difference was not significantly different. Australia's Year 8 performance was statistically similar to that of eight other countries. It was also below nine countries, again including Chinese-Taipei and the US, but higher than that of 31 other participating countries.



READING THE GRAPHS

Each country's results are represented in horizontal bars with various colours. On the left end of the bar is the 5th percentile – this is the score below which 5 per cent of the students have scored. The next line indicates the 25th percentile. The next line at the left of the white band is the lower limit of the confidence interval for the mean – i.e. there is 95 per cent confidence that the mean will lie within this white band. The line in the centre of the white band is the mean. The lines to the right of the white band indicate the 75th and 95th percentiles.

Figure 4 shows the percentage of students from Australia, Chinese-Taipei and the US who reached the four benchmark levels in mathematics.

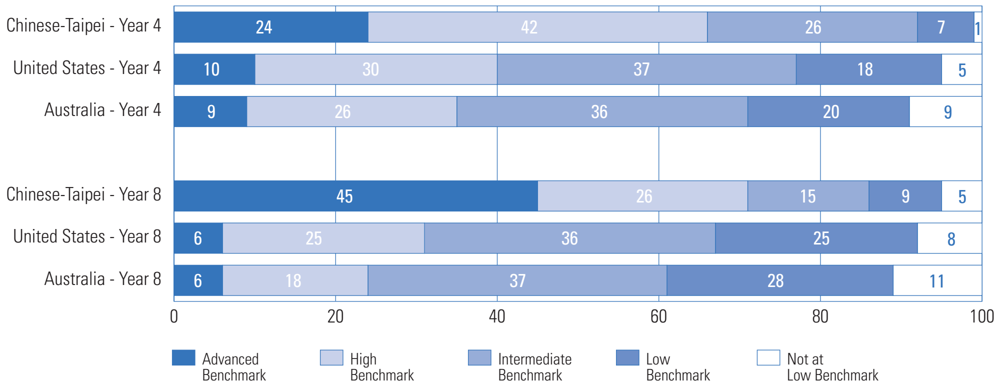


Figure 4 Percentage of students reaching the international benchmarks in Year 4 and Year 8 mathematics

Figure 4 shows that Chinese-Taipei had the most positive results; that is, they had the highest percentage of students achieving the advanced benchmark, and the lowest percentage of students not reaching the low benchmark, for both Year 4 and Year 8 mathematics. At both year levels, Australia and the US shared the same or similar percentages for the number of students at the advanced benchmark; however, while 35 per cent of Year 4 and 24 per cent of Year 8 Australian students achieved at the high international benchmark, Australian students had the highest percentages (compared to these countries) of students not achieving the low benchmark for Year 4 and Year 8 mathematics.

In addition to average achievement levels, benchmark status was also related to each achievement item contained in the TIMSS 2007 assessment. In other words, every item was linked to attainment of one of the four benchmarks. The associated benchmark for each item discussed in this report is listed during the item analyses.

How can TIMSS results inform pedagogy for classroom teachers?

The main purpose of this report is to present TIMSS 2007 mathematics results in a way that can inform pedagogy. The report explores students' responses to a selection of mathematics items and then considers what these responses might indicate about students' level of understanding for a particular item and its content area. By breaking down the results at the item level, teachers can ascertain whether the mistakes typically made by students in the sample are also mistakes made by their own students.

Not all items included in the TIMSS 2007 project are available to the public. The selection of items discussed in this report was made from the group which have been publicly released. The CD included with this report contains all of the TIMSS 2007 released items so that teachers may see the types of questions students completed when they participated in the project.

Using students' answers as evidence of their understanding

Item analysis of the TIMSS 2007 results can inform pedagogy because we assume that students' answers can be used as evidence of their understanding of mathematics concepts. For instance, items presented via a multiple choice format offer a good opportunity to evaluate students' understanding in large scale studies like TIMSS. The series of options provided in a multiple-choice question contain the correct answer in addition to a set of distracter options. Some of the distracters represent answers that students will obtain if they have a basic understanding of the area, some if students have an intermediate understanding of the material and some are extreme answers that are likely picked if students guess. Therefore, if a majority of students select a distracter as the answer for an item, it is possible to discuss the average level of understanding that Australian Year 4/Year 8 students are operating at. This discussion is possible due to the fact that TIMSS is a large scale study that assesses the achievement of a representative student sample.

Another important factor to consider is the percentage of students that omit giving an answer for an item. There are two logical explanations for this. The first is that the information is missing at random; in other words, students missed the item by accident. The second explanation is that the information is missing for a reason. In this case, educators must ask – did students omit the answer because (i) they believed they did not have ability to obtain a solution or (ii) they did not want to put in the effort required to complete the task? This report will assume that it is the former, perceptions of doubt in relation to ability, that explains the percentage of omitted answers for an item, rather than a lack of effort, or because the information was missing at random.

Types of items and item analysis

Every item included in the TIMSS 2007 project had several assessment characteristics. Each corresponded to a curriculum area (*mathematics* or *science*), a content area (e.g. *algebra* or *chemistry*) and a cognitive skill (*knowing*, *applying* or *reasoning*)[1]. The mode of item presentation also varied. Items were either *multiple-choice* questions or questions requiring a *constructed–response*. The majority of items discussed in this report were of the multiple-choice format.

The following sections of this report present the item analyses for a selection of TIMSS 2007 results. Each item is presented along with a set of percentages. These percentages might include the number of students who obtained the correct/incorrect solution, the number of students who selected a particular distracter option, the number of students who omitted giving the answer to an item, or the percentage of gender difference in the number of correct answers given (e.g. if girls performed better than boys on average, by how large a percentage?). Rather than providing all these figures for every item, a set of percentages were selected for each that reflected the most

1 Note that items assessing the last of the cognitive domains, reasoning, are not considered in this report.

significant trend for that item. Note that all percentages discussed have been rounded to the nearest whole per cent except in the case of figures below 0.5%.

Part of the TIMSS 2007 project also included surveying the teachers of the participating students. Part of this survey asked these teachers to rate whether topics assessed in the TIMSS items had been (i) "mostly taught before this year"; (ii) "mostly taught this year"; or (ii) "not yet taught or introduced". For each item analysed in this report, the percentage of students who had teachers that rated 'yes' to the first two categories is given. Thus, we report the proportion of students that, according to their teachers, had been exposed to the relevant topics prior to their participation in TIMSS.

What can TIMSS tell us about Year 4 mathematics?

Five mathematics items from the set presented to Year 4 students are discussed in this report. The first two relate to the *number* area. In addition, they both assessed *knowing*; that is, students' knowledge and use of skills typical of the number domain.

Item 1 presented a multiplication problem and was associated with the *high benchmark*. Ninety-nine per cent of Australian students had teachers who reported having taught computation with whole numbers.

M02_03

1

Multiply:

53×26

Answer: ______________

M041278

On average, 86% of Australian Year 4 students attempted to answer this question; however, only 9% obtained the correct answer, compared with 50% of US students and 88% of students from Chinese-Taipei. The number of Australian students that avoided or omitted answering the item was 14%, compared to 2% of US and 1% of Chinese-Taipei students. Both the number of incorrect answers and the level of omitted answers are concerning figures. They suggest that, on average, Australian Year 4 students had not mastered the ability to multiply 2-digit numbers together, whereas half of US students and a majority of Chinese-Taipei students had.

The second item from the Year 4 *number* domain assessed students' understanding of fractions via a multiple-choice question and was from the pool of *advanced benchmark* items. Fifty-three per cent of Australian students had teachers who stated that the topic of equivalent fractions had been taught prior to the TIMSS 2007 project.

M04_03

2

Which fraction is equal to $\frac{2}{3}$?

Ⓐ $\frac{3}{4}$

Ⓑ $\frac{4}{9}$

Ⓒ $\frac{4}{6}$

Ⓓ $\frac{3}{2}$

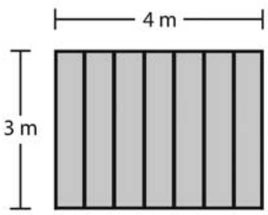
M041069

All three countries showed poor performance on this item with achievement rates for Australian students being the lowest. On average, 21% of Australian students selected option C, that the equivalent fraction was *4/6*. Forty-five per cent of US students and 47% of students from Chinese-Taipei provided the same response. The most important finding for this item was the distracter option that the majority of students selected. In general, 76% of Australian students answered the item incorrectly. Fifty-four per cent identified *3/2* as the equivalent fraction to *2/3*. The US and Chinese-Taipei also had large numbers of students selecting this distracter (29% and 33%, respectively). This illustrates a deficiency in students' understanding of the numerator and denominator. It might also reflect the fact that approximately half of participating Australian students had not been exposed to the topic before.

Items 3 and 4 assessed Year 4 students' *geometry* skills. Item 3 was another linked to the *advanced benchmark*. This multiple-choice question was designed to examine students' knowledge (*knowing*) of two-dimensional shapes.

M04_08

3



Patrick is painting one side of a fence. The fence is 4 metres long and 3 metres high. What is the area that Patrick has to paint?

Ⓐ 4 square metres

Ⓑ 7 square metres

Ⓒ 12 square metres

Ⓓ 14 square metres

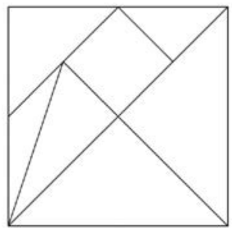
M041152

Compared with 48% of US and 82% of Chinese-Taipei students, only 28% of Australian students selected *12 square metres* as the answer. Again, the most significant aspect of the results for this item was the distracter most commonly picked by students. Forty-two per cent of Australian Year 4 students selected *7 square metres* as the solution, showing that they most likely added the length and width of the rectangle together in order to find the area. Alternatively, they may have simply counted the number of vertical bars in the diagram. Twenty-eight per cent of US and ten per cent of Chinese-Taipei students also selected *7 square metres*. Another interesting finding for this item related to gender differences for the percentage of correct responses. Eight per cent more girls than boys in the US and Chinese-Taipei, on average, got this item correct. For Australian students, on the other hand, the gender difference was smaller and in favour of boys, who had a 5% higher correct response rate. The teachers of 69% of Australian students stated that the subject of calculating area had been addressed in class.

The second item assessing *geometry*, Item 4, illustrated a more positive result for Australian Year 4 students; however, this item was linked to the *low benchmark*. Students were required to 'mark' their response rather than select the correct solution for a set of multiple options. A square was partitioned into seven triangles and students were asked to mark which two of these were the same size and shape. This item represented an opportunity to demonstrate the *application* cognitive skill in the domain of geometry.

M01_09

4 The square is cut into 7 pieces. Put an X on each of the 2 triangles that are the same size and shape.



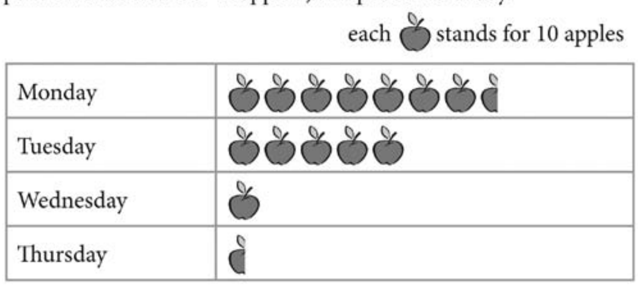
M031271

Eighty-five per cent of Australian students correctly marked the two triangles that formed the right-bottom corner of the square. A similar 86% of US students and 81% of Chinese-Taipei students also answered correctly. However, Chinese-Taipei students also showed the largest rate of omission for this item at 16%, whereas 8% and 10% of Australian and US students, respectively, omitted this item. Sixty-nine per cent of Australian students had teachers who reported previously teaching the topic of geometric shapes and their properties.

The last Year 4 mathematics item investigated the *data display* content area and called for students to read and interpret a graph as part of the *knowing* cognitive skill. Ninety-four per cent of Australian students had teachers who reported that the skills of reading information from tables, pictographs, bar graphs or pie charts had been covered in their classrooms. Item 5 was associated with the *high benchmark*.

M04_12

5 The graph shows the number of apples John picked each day.



On which day did John pick 5 apples?

Ⓐ Monday
Ⓑ Tuesday
Ⓒ Wednesday
Ⓓ Thursday

M041186

Australian students' performance on this item sat between that of the other two comparison countries. Surprisingly, Chinese-Taipei students had the lowest percentage of correct answers, with just 57% answering correctly. Seventy-one per cent of US students and 61% of Australian students selected the correct alternative. The most commonly chosen distracter option was option B, indicating that many students did not interpret the legend correctly and just selected the alternative showing five apple pictures. Interestingly, 8% of US students failed to answer this question compared to 2% of Australian students and 1% of students from Chinese-Taipei.

What can TIMSS tell us about Year 8 mathematics?

The first two of the five Year 8 mathematics items presented assessed *algebra* and the *knowing* cognitive domain. Item 6, linked to the *high benchmark*, examined students' ability to collect like terms via a multiple-choice question. Results indicated that the majority of Chinese-Taipei students had a firm understanding of this concept. In general, Australian and US students did not.

M02_06

6

Which is equivalent to $4x - x + 7y - 2y$?

Ⓐ 9

Ⓑ $9xy$

Ⓒ $4 + 5y$

Ⓓ $3x + 5y$

M042199

Ninety per cent of Chinese-Taipei students answered correctly compared with 51% and 65% of Australian and US students, respectively. Seventy-seven per cent of Australian students had teachers who said that algebraic simplification was a topic that had been addressed prior to the TIMSS 2007 project. This percentage was also relevant for Item 7. This item assessed students' mastery of algebra at a deeper level as it called for the use of two algebraic skills. It was also from the pool of *advanced benchmark* items. Successful completion of this question called first for expansion, and then the collection of like terms.

M04_07

7

Which of these is equal to $2(x + y) - (2x - y)$?

Ⓐ $3y$

Ⓑ y

Ⓒ $4x + 3y$

Ⓓ $4x + 2y$

M042239

The expansion component of this question was particularly challenging as it required recognition that a -1 should be multiplied with every term within the second, bracketed expression. Thus, the last term to result from the expanding procedure should be y, following the multiplication of -1 and $-y$. Only 16% of Australian students selected the correct solution, $3y$, from the four options. This was a similar figure to the US' 19%, while 72% of Chinese-Taipei students answered correctly.

By examining the response rate for distracter options, it is possible to identify students who *incorrectly* completed the expanding component of the item but were able to *correctly* collect like terms with the consequential terms they obtained. The working of these students would have produced y as the final solution. Table 3 shows the percentage of students who selected this distracter for Item 7 along with the rate of responses for the other distracter options. Students who selected either of the remaining two distracter options ($4x+3y$ or $4x+2y$) demonstrated a poor understanding of both algebraic components of Item 7.

Table 5 Percentage of Year 8 students that selected incorrect distracter options according to country

	Item: 2(x + y) – (2x – y)		
	y	*4x + 3y*	*4x + 2y*
Australia	28	20	30
Chinese-Taipei	14	7	7
US	24	20	34

The shaded cells of Table 5 represent the distracter that was selected by the majority of students in a country. Table 5 shows that a larger proportion of the Chinese-Taipei students who answered incorrectly selected the distracter that showed they had a partial understanding of the algebra required for the item. On the other hand, the distribution for Australia and the US was more spread out, and the most commonly selected distracter was one that reflected poor algebraic understanding. For both Year 8 algebra items (Item 6 and 7), five per cent of Australian students omitted giving an answer. This number was much lower for the US and Chinese-Taipei and tended to be one per cent or lower except for US students in relation to Item 7 where 2% of students failed to answer the question.

Australian students' responses to Item 8 proved to be more encouraging. This question measured the ability to *apply* mathematical skills characteristic of the *number* domain, and was an item linked to the *intermediate benchmark*.

M01_06

8

One year a company reported selling 1426 tonnes of fertiliser. The following year the company sold 15 percent less fertiliser. Which is the closest approximation to the number of tonnes of fertiliser sold in the second year?

Ⓐ 200

Ⓑ 300

Ⓒ 1200

Ⓓ 1600

Ⓔ 1700

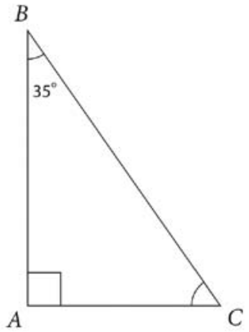
M022057

Seventy-three per cent of Australian students answered this item correctly, demonstrating an understanding of percentages and the ability to apply this knowledge to a contextualised problem. The teachers of 90% of Australian students stated that the conversion of percentages to fractions or decimals had been previously taught. Seventy per cent of US students chose the correct alternative for Item 8. Chinese-Taipei students had the lowest percentage of correct responses at 65%. A commonality across the three countries for this item was the pattern in terms of gender. The percentage of correct responses given by boys was higher by 6% in Chinese-Taipei, 8% in Australia and 11% in the US.

Item 9 assessed students' application *(applying)* of *geometry* skills. Like Item 8, it was an *intermediate benchmark item*. To solve the problem correctly, students needed to know the angle sum of a triangle and the symbol used to denote 90 degree angles. They then had to use this knowledge to find out the size of the unknown angle. Eighty-six per cent of Australian students had teachers who stated that the properties of geometric shapes (including triangles) had already been covered in the classroom.

M03_14

9



What is the measure of angle C in the triangle above?

Ⓐ 45°
Ⓑ 55°
Ⓒ 65°
Ⓓ 145°

M032579

Chinese-Taipei students, on average, showed the best performance on this item with 87% of students choosing *55 degrees* as their answer. Next was Australia with 60% of students correct and lastly was the US at 55%. Twenty-four per cent of Australian students and 28% of US students selected the distracter, *45 degrees*.

The last Year 8 mathematics item, Item 10, was from the *data and chance* domain and involved the application *(applying)* of skills typically developed in this content area. This item, representing the *high benchmark*, drew on students' knowledge of probability as well as an understanding of equivalent fractions.

M01_07

10

A bowl contains 36 coloured beads all of the same size: some blue, some green, some red, and the rest yellow. A bead is drawn from the bowl without looking. The probability that it is blue is $\frac{4}{9}$. How many blue beads are in the bowl?

Ⓐ 4
Ⓑ 8
Ⓒ 16
Ⓓ 18
Ⓔ 20

M022257

Only 45% of Australian students identified *16* as the correct answer, compared with 50% of US students and 83% of students from Chinese-Taipei. Twenty per cent of Australian students and 25% of US students selected *4* compared with 9% of Chinese-Taipei students. Another important pattern for Item 10 was the 15% higher correct response rate for boys in Australia. Gender differences were also apparent with US students, again favouring boys but at a smaller 5%. The difference in favour of boys for Chinese-Taipei was only 0.1%. Interestingly, only 35% of Australian students had teachers that reported that they had covered the topic of problem solving via the use of probabilistic outcomes.

Summary and general trends

The items discussed in this report represent a small selection of the total item inventory that was part of the TIMSS 2007 project; however, investigation of these items illustrated areas of strength and (particularly) weakness for Australian students that warrant educators' consideration. While it is difficult to identify trends based on a small sample of items, there were also some central themes that emerged through the item analysis conducted.

At the national level

The five Year 4 mathematics items reviewed students' skills in number, geometry and data. Australian students performed well on the item assessing their understanding of shapes but their achievement was poorer for other areas, especially in items related to multiplication, fractions and area.

For Year 8 mathematics, the five items investigated students' understanding of algebra, number, geometry and data. Items assessing algebra revealed a particular area of weakness for Australian students as did a data question that contained components of probability and fractions.

For some of the items discussed the percentage of answers omitted was quite large. Avoidance of these items is an issue of concern whether it was due to poor competence beliefs or lack of effort. Lastly, larger gender differences for the rate of correct responses tended to favour boys. This trend meets with the general TIMSS 2007 finding that boys outperformed girls in mathematics.

At the international level

For the most part, the international comparisons made between Australia, the US and Chinese-Taipei served to highlight areas of weakness for Australian students. There was an obvious gap between the understanding of Australian students and their Chinese-Taipei counterparts. Furthermore, where Australian and US students previously achieved at similar levels (see TIMSS 2003 results), in TIMSS 2007 the US outperformed Australia in almost all the items reviewed.

On the other hand, the international comparison made also illustrated the trap of simply considering average level results for a country. In three of the ten items reviewed, Chinese-Taipei was outperformed by either the US or Australia. This finding does not diminish the high achievement levels of the Chinese-Taipei students who participated in TIMSS 2007 but emphasizes that, while they were ranked in the top three for all grade/curriculum areas, Chinese-Taipei students still demonstrated skill deficiencies in some areas.

Interestingly, gender differences varied across countries. Some domains showed large gender differences for some countries but negligible ones for others (e.g. Year 8 mathematics).

Informing pedagogy

The primary aim of this report was to provide an informative review of Australian students' performance in TIMSS 2007. This analysis was conducted with the hope that teachers might reflect on the results and that this might be helpful to their classroom teaching initiatives.

Martinez[2] (2001) pointed out that it is not useful to consider TIMSS as an international achievement competition. Rather it is a "compendium of curricular data, educational cultures, teaching and learning styles, and assessment techniques" (p.114). The TIMSS 1999 Video Study of Eighth Grade Mathematics Teaching demonstrated both the similarities and discrepancies in teaching amongst eight participating countries that included Australia and the US. For instance, Australia and Japan spent more time practicing new content in the classroom than the US who devoted more time to reviewing material. Thus, the strengths and weaknesses of students' responses highlighted in this report were likely due to many different factors. However, with more awareness of students' understanding in different curriculum areas, educators can develop learning strategies that suit their particular teaching styles and unique educational contexts.

More information!

Included with this report is a CD which contains all of the released mathematics items for Year 4 and Year 8, along with the scoring guide for each item. Teachers can use these to see how items on international assessments are constructed and scored, and use the formats for their own testing.

Also on the CD are the item almanacs for all of the released items. These show, for each item, the number of students who attempted the item, the percentage of students who responded to each of the various marking codes, the percentage of students who omitted the item and the percentage of students who did not reach it. Also included are the percentage of students overall who gave a correct answer, and the percentage of boys and girls in each country giving a correct answer. We encourage teachers to explore these statistics for themselves.

Further information and all reports on all TIMSS assessments is available from the TIMSS website, at www.acer.edu.au/timss.

2 Martinez, J. (2001) Exploring, inventing, and discovering mathematics: A pedagogical response to TIMSS. *Mathematics Teaching in Middle School, 7 (2)*, 114-120.

www.acer.edu.au